Maths Calculation Policy

John Clifford School

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'Mathematics is a creative and highly inter-connected discipline essential to everyday life. A high- quality mathematics education provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.' - **National Curriculum, 2014**

Maths Calculation Policy

At John Clifford School, our planning and teaching of mathematics is drawn from the objectives set within the National Curriculum. This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added including a set of expected layouts for standard written methods that will be used wherever taught. It is a working document and is amended as necessary. Many variations have been included to provide teachers with a range of tools and strategies to support children in their grasp of number and calculation. To ensure consistency for pupils, it is important that the mathematical language used in Maths lessons reflects the vocabulary used within this policy.

This policy has been designed to teach children through the use of concrete, pictorial and abstract representations. The Concrete, Pictorial, Abstract (CPA) approach is a highly effective approach to teaching that develops a deep and sustainable understanding of Maths in pupils. CPA was developed by American psychologist Jerome Bruner. Progression within each area of the curriculum is in line with the programme of study in the 2014 National Curriculum.

Concrete Learning:

Concrete is the 'doing' stage. During this stage, children use concrete objects to model problems. This brings concepts to life by allowing children to experience and handle physical (concrete) objects. With CPA teaching, every abstract concept is first introduced by using physical, interactive, concrete materials.

For example, if a problem involves adding pieces of fruit, children can first handle actual fruit. From there they can progress to handling abstract counters or cubes which represent the fruit.

Pictorial Learning:

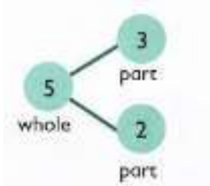
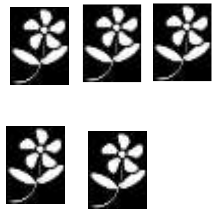
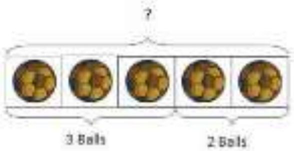
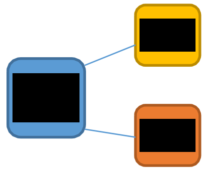
Pictorial is the 'seeing' stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make metal connections between the physical object they just handled and abstract pictures, diagrams or models that represent the objects from the problem.

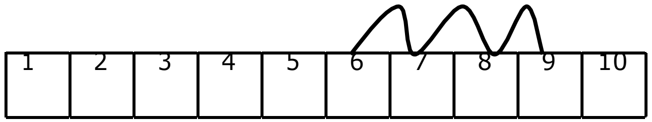
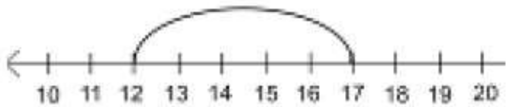
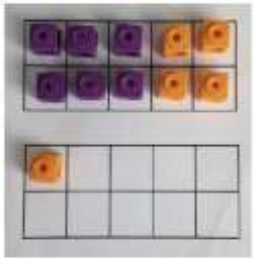
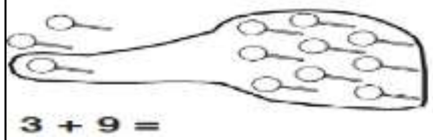
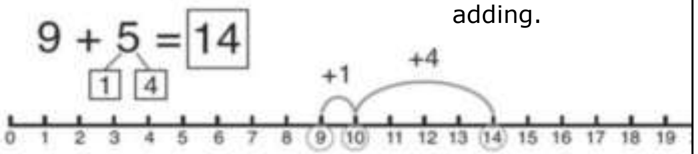
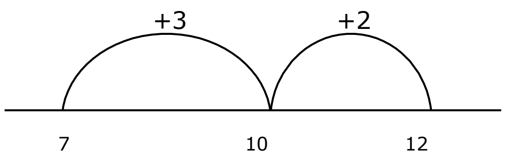
Building or drawing a model makes it easier for children to grasp difficult abstract concepts (for example, fractions). It helps children to visualize abstract problems and make them more accessible.

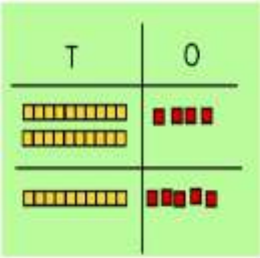
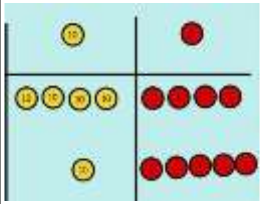
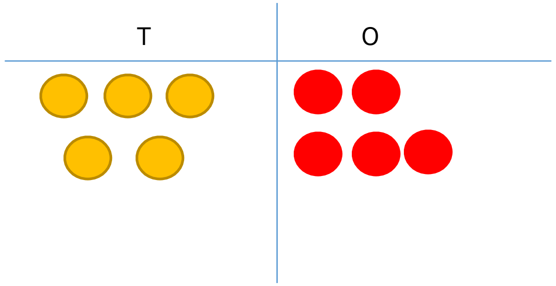
Abstract Learning:

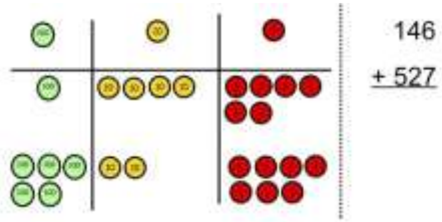
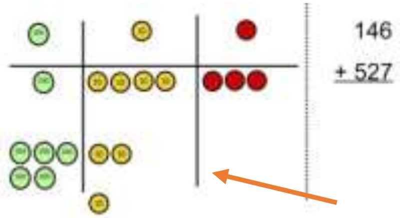
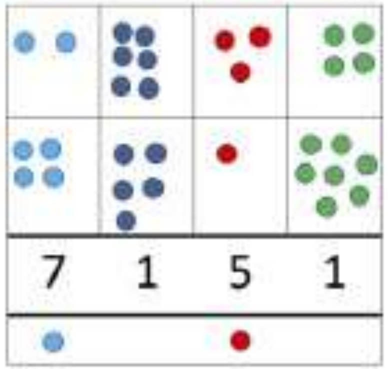
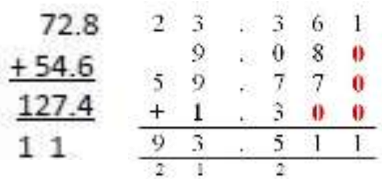
Abstract is the 'symbolic' stage, where children use abstract symbols to model problems. Children will not progress to this stage effectively until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, +, -, x, ÷) to indicate addition, subtraction, multiplication or division.

Progression in Calculations
Addition

Method	Concrete	Pictorial	Abstract
Counting a set of objects. This can include counting using fingers. Subitising (the ability to quickly look at a small number of objects and know how many there are without counting) underpins this skill.	 		 Children relate the number of objects to the numeral.
Combining 2 separate amounts to make 1 whole amount.	  For 4 + 3, count out 4 cubes then 3 more and group them together to see what they have altogether. This can also be represented in a bar. E.g. for 8 + 1: 	   Use pictures to add two numbers together as a group or in a bar. 	 Use the part-part whole diagram as shown above to move into the abstract. 4 + 3 = 7 10 = 6 + 4 Although number sentences are recorded in the concrete and pictorial methods, the abstract method sees the calculation carried out without the use of concrete or pictorial aids.

Start at the bigger number and count on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	Counting on in jumps of 1 using a number line with numbers on it. For 6 + 3 = 9:  This can also be done in bigger jumps or 1 big jump to find the answer. For 12 + 5 = 17: 	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
'The Magic 10' Regrouping to make 10 so that the calculation is easier.	Regroup 9 + 3 into 10 + 2 before adding together:   6 + 5 = 11 Start with the bigger number and use the smaller number to make 10.	 Use pictures or a number line. Regroup or partition the smaller number to make 10 before adding.  Children move on to using an 'empty number line'. E.g. 7 + 5 becomes 7 + 3 + 2 	7 + 5 = 7 + 3 + 2 = 12 If I have seven, how many of my 5 do I need to add to make 10. How many more do I still need to add on?

Column addition without regrouping	24 + 15 = 39  Partition the numbers into tens and ones using Dienes blocks. Add together the ones first then add the tens. Finally add the 2 totals together. 44 + 15 = 59  Move onto using place value counters.	After practically using the Dienes blocks and place value counters, children can draw the counters to help them to solve additions. 32 + 23 = 55  	21 + 42 = 21 + <u>42</u> Record the calculation vertically adding the column of ones then the column of tens.

Column addition with regrouping	Make both numbers with place value counters.  In this case, adding the ones gives us 13 which is made up of 10 and 3.  Exchange 10 of these ones for one 10 and add it together with the other tens. Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column if needed. This can also be done with Dienes equipment to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  	Begin by partitioning the numbers: For 76 + 47 70 + 6 40 + 7 ~~110 + 13~~ = 123 Move on to clearly show the exchange below the addition: 70 + 6 40 + 7 120 + 3 = 123 10 This then becomes the compact method where numbers aren't partitioned but exchanges still take place: 76 + 47 123 1 1 As the children move on, introduce decimals with and without the same number of decimal places. Money can also be used here.  N.B. Exchanged digits need to be recorded below the line when adding.

Column addition

Expected layout

1) 2 9 3 + 4 5

	H	T	O
	2	9	3
+	0	4	5
	3	3	8
	1		

- ✓ Line up **digits** correctly according to **place value** with one digit in each box
- ✓ You may use additional 0s as **place holders** to help you carry out the calculation
- ✓ Work from the **right**
- ✓ **Exchanged** numbers are ‘**carried**’ into the next column along for addition

Column addition with decimals

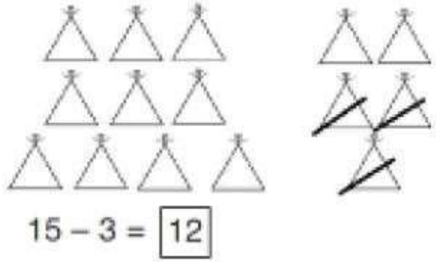
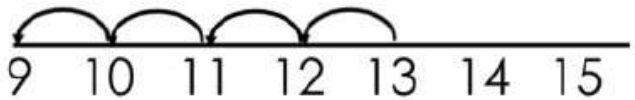
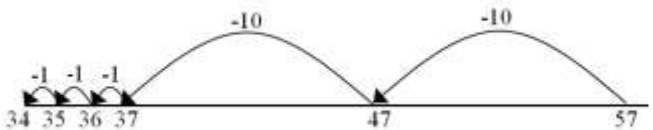
Expected layout

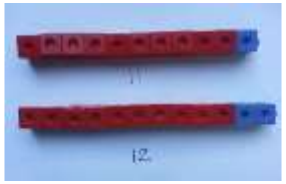
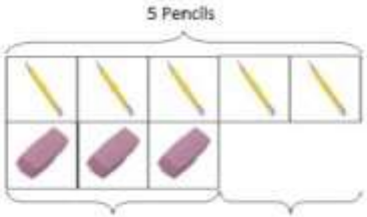
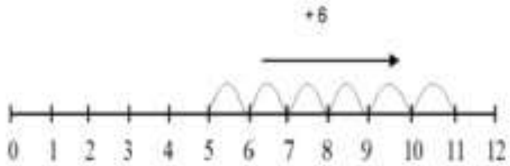
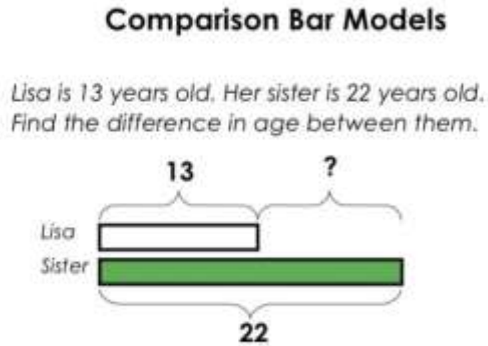
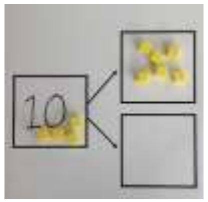
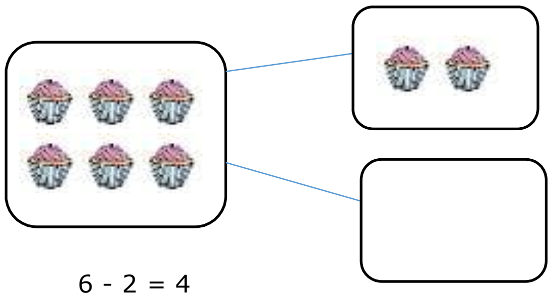
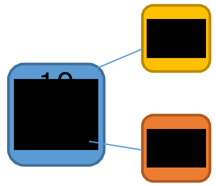
1) 3.125 + 0.5

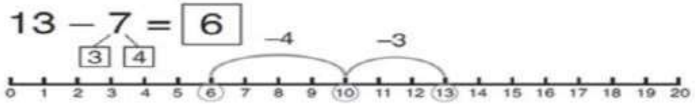
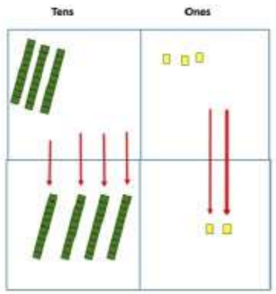
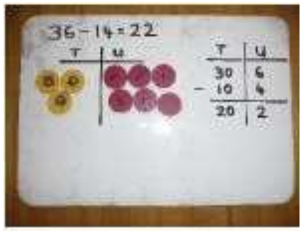
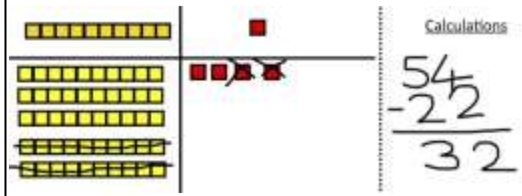
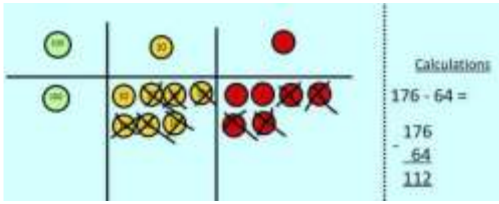
	H	T	O	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
			3.	1	2	5
		+	0.	5	0	0
			3.	6	2	5

- ✓ Line up **digits** and **decimal points** correctly with one digit in each box
- ✓ You may use additional 0s as **place holders** to help you carry out the calculation
- ✓ Use column addition as usual, working from the right
- ✓ Carry the **decimal point** straight down into the answer

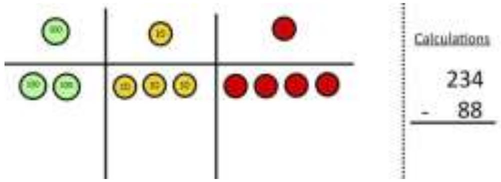
Subtraction

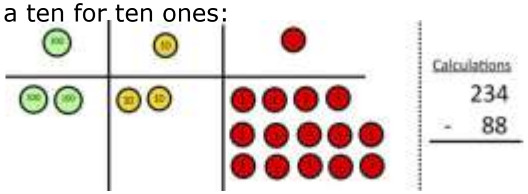
Method	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc. to show how objects can be taken away.  6 - 2 = 4 	Cross out drawn objects to show what has been taken away. 4 - 2 = 2   	18 -3= 15 8 - 2 = 6 Although number sentences are recorded in the concrete and pictorial methods children are introduced to them on their own while encouraging them to mentally take away ones.
Counting back	Make the larger number in the subtraction. Move the beads along the bead string and count backwards in ones. 13 - 4  Use counters and move them away from the group counting backwards as they e.ch one is moved away. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	For 13 - 4, put 13 in your head and count back 4. What number are you at? Use your fingers to help.

Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference.  Use basic bar models with items to find the difference.	 11 - 5 = 6 Count on to find the difference. Draw bars to find the difference between 2 numbers.  	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part whole model.  6 - 2 = 4	 Move to using numbers within the part whole model.

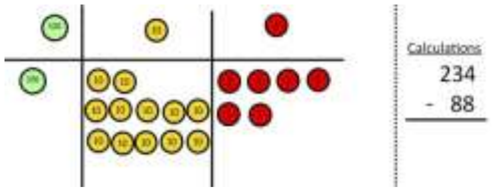
Make 10	14 - 5 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Count back 3 to reach 10. Then count back the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 - 8= How many do we take off to reach the previous 10? (6) How many do we have left to take off? (2)
Column method without regrouping	75 - 42  Use Dienes blocks to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first.  	  Draw the Dienes or place value counters alongside the written calculation to help show working.	Partitioned numbers are written vertically: For 54 - 22 Tens Ones 50 4 - 20 2 30 + 2 = 32 This will lead to a clear written column subtraction: 54 - <u>22</u> 32

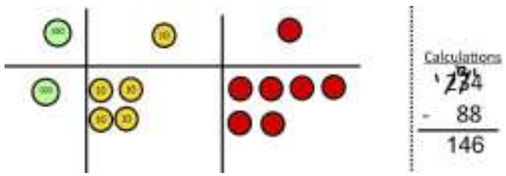
<table>
<tr>
<td>Column method with regrouping</td>
<td>
Use Dienes first then move to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

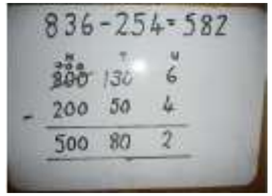
Make the larger number with the place value counters


Start with the ones. I can’t take away 8 ones. I need to exchange a ten for ten ones:


Now I can subtract 8 ones from 14.

Next look at the tens. I can’t take away 8 tens. I need to exchange a hundred for 10 tens:


Now I can take eight tens from the 12 tens and complete the subtraction.


Show children how the concrete method links to the written method alongside their working. Cross out the numbers when exchanging and show where we write our new amount.
</td>
<td></td>
<td>


Children can start their formal written method by partitioning the number into clear place value columns.


Moving forward the children use a more compact method.

This will lead to an understanding of subtracting any number including decimals.
$$\begin{array}{rrrrr} & ^{5} & ^{12} & & \\ 2 & \cancel{6} & \cancel{3} & . & {}_{1}0 \\ - & 2 & 6 & . & 5 \\ \hline 2 & 3 & 6 & . & 5 \end{array}$$
</td>
</tr>
</table>

Column subtraction

Expected layout

2) 4 7 2 - 6 5

	H	T	O
	4	$\cancel{7}^{6}$	$^{1}2$
-	0	6	5
	4	0	7

- ✓ Line up **digits** correctly according to **place value,** with one digit in each box, remembering to take the smaller number away from the greater number
- ✓ You may use additional 0s as **place holders** to help you carry out the calculation
- ✓ Work from the **right**
- ✓ **Exchanged ('borrowed')** numbers are clearly and neatly indicated

Column subtraction with decimals

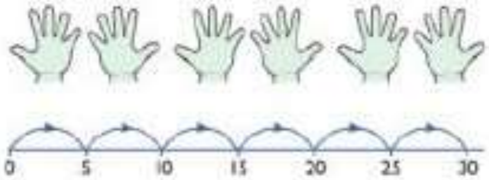
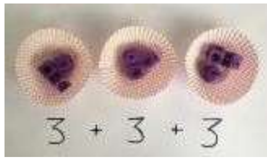
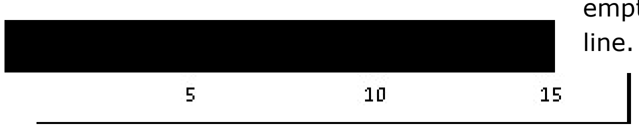
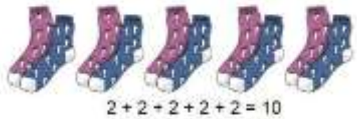
Expected layout

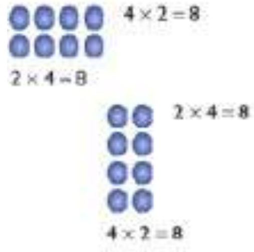
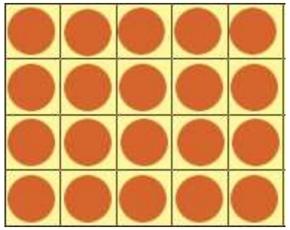
2) 1 2 . 5 - 6 . 2 5

	H	T	O	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
		1	2 .	5	0	
-			6 .	2	5	
			.			

- ✓ Line up **digits** and **decimal points** correctly with one digit in each box
- ✓ You may use additional 0s as **place holders** to help you carry out the calculation
- ✓ Use column subtraction as usual, working from the right
- ✓ Carry the **decimal point** straight down into the answer

Multiplication

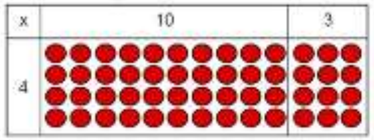
Method	Concrete	Pictorial	Abstract
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count out loud in multiples of a number. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Repeated addition	   Use different objects to add equal groups.	5 + 5 + 5 = 15 5 5 5 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 Repeated addition can be shown on a labelled or empty number line.  Begin to relate repeated addition to multiplication using 'lots of' e.g. 3 lots of 5 = 15	Write addition sentences to describe objects and pictures.  This then leads to writing related multiplication sentences e.g. 2 x 5 = 10

Arrays- showing commutative multiplication	 Create arrays using counters / cubes to show multiplication sentences. 4 x 6 = 24 Begin to look at arrays in different orientations to make the link between, for example, 5 x 3 = 15 and 3 x 5 = 15 (commutativity) 	Draw arrays in different rotations to find commutative multiplication sentences.  Link arrays to area of rectangles. 	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Grid Method

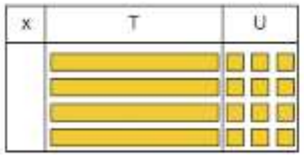
Show the link with arrays to first introduce the grid method.

4 x 13



4 rows of 10
4 rows of 3

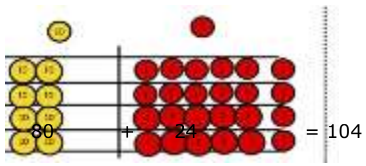
Move on to using Dienes to move towards a more compact method.



4 rows of 13

Move on to place value counters to show how we are finding groups of a number.

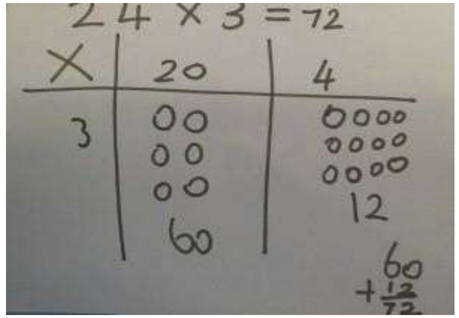
4 x 26



We are multiplying by 4 so we need 4 rows with each containing 26.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying 2-digit by 1-digit numbers showing the addition alongside the grid.

×	30	5
7	210	35

210 + 35 = 245

Moving forward, multiply 2, 3 and 4-digit numbers showing the different rows within the grid method.

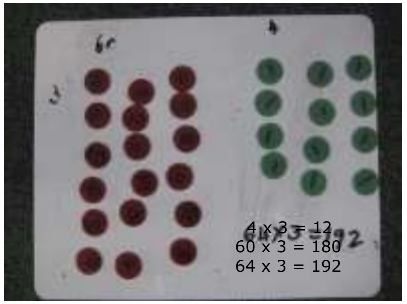
13 x 28

X	20	8
10	200	80
3	60	24

  280
+ 84
  364
  1

<table>
<tr>
<td>Short & Long Multiplication

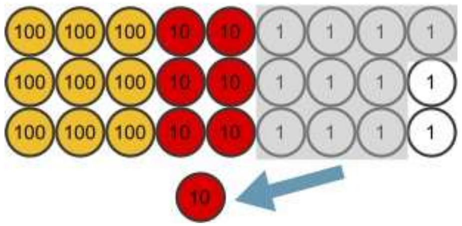
(Column multiplication)</td>
<td>Children can continue to be supported by place value counters for carrying out column multiplication.
They can partition and record each calculation vertically.



It is important to get into the habit of multiply the ones first and note down their answer followed by the tens which they note below.

The idea of exchanging will support them in moving on to a more compact method:

3 x 324

</td>
<td>As with stage 4, children can represent the work they have done with place value counters in a way that they understand.
They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking.</td>
<td>As with the grid method, numbers of more than one digit are partitioned but this time the calculation is recorded vertically. To support them, children need to write out what they are solving next to their answer.

For 38 x 7

38
X 7
56   8 x 7
210   30 x 7
266

Remind the children about the importance of lining up their numbers clearly in columns.

This then moves to the more compact method of short multiplication:

For 56 x 27

56
x 27
392 56 x 7
1120 56 x 20
1512
1

Start by multiplying the ones digit, recording the last digit of the answer in the answer line but exchanging any tens and putting them under the tens column to be added on after multiplying the tens digit. Again, the last digit in the answer is recorded in the answer line and any hundred are exchanged, this time to the hundreds column, and so on.</td>
</tr>
</table>

Short multiplication

Expected layout

1) 7, 0 5 1 x 4

M	100 Th	10 Th	Th	H	T	O
			7,	0	5	1
		x				4
		2	8,	2	0	4
		2		2		

- ✓ One digit in each box
- ✓ Digits correctly lined up according to place value
- ✓ Work from the right
- ✓ **Exchanged** numbers are '**carried**' into the next column along for addition

Long multiplication

Expected layout

2)

	M	100 Th	10 Th	Th	H	T	O
				1,	2	5	5
			x			3	1
(1255 x 1)				1	2	5	5
(1255 x 30)		+	3	7^1	6^1	5	0
			3	8,	9	0	5
					1		

- ✓ One digit in each box
- ✓ Digits correctly lined up according to place value
- ✓ Work from the right
- ✓ Two row answer box
- ✓ **0** used as a **place holder** in second row
- ✓ Both rows are added up to find the answer
- ✓ **Exchanged** numbers are '**carried**' into the next column along for addition

Multiplication of decimals

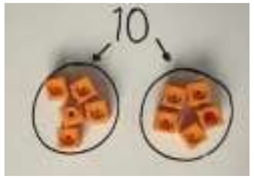
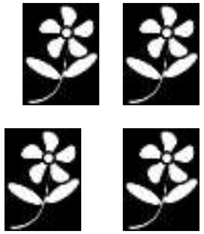
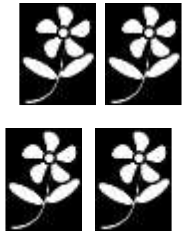
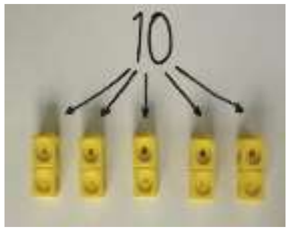
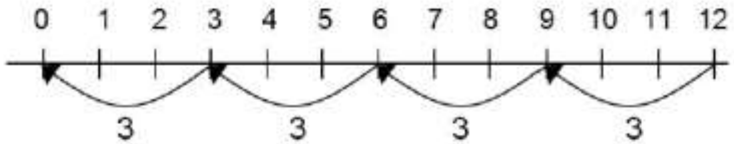
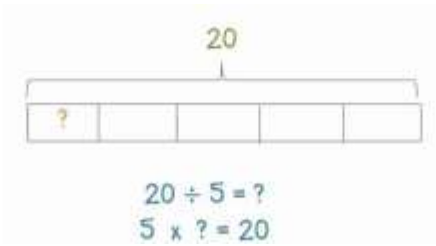
Expected layout

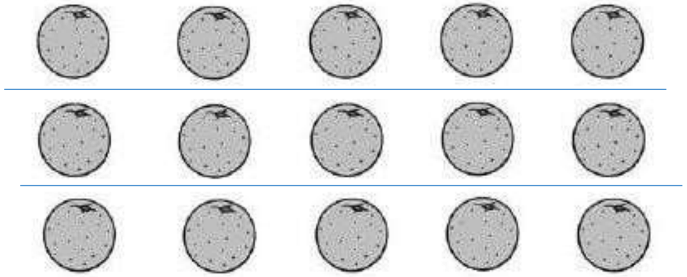
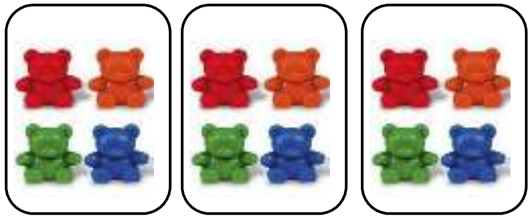
2)

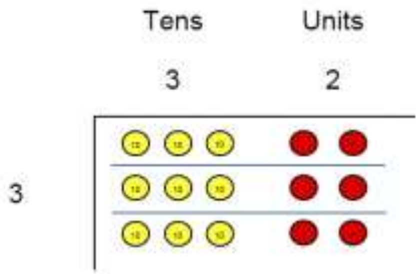
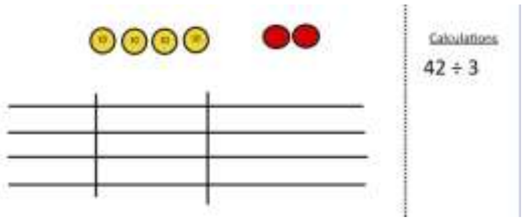
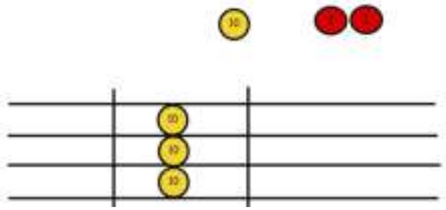
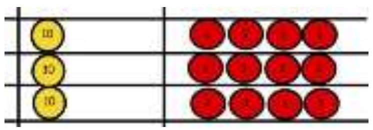
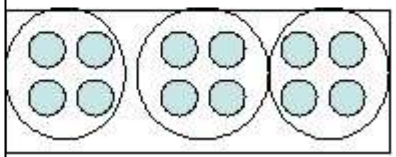
M	100 Th	10 Th	Th	H	T	O
			1	2 ▪	5	5
		x			3 ▪	1
			1	2	5	5
	+	3	7^1	6^1	5	0
		3	8 ▪	9	0	5
				1		

- ✓ One digit in each box
- ✓ Note that decimal points **do not** need to be correctly lined up
- ✓ Work from the right, using the usual written multiplication method
- ✓ The answer will have the same number of decimal places as the multiplied numbers combined (in this case, 3 decimal places).

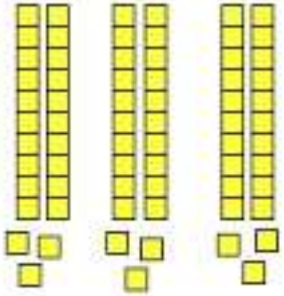
Division

Method	Concrete	Pictorial	Abstract
Sharing objects equally	 I have 10 cubes, can you share them equally in 2 groups? 	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  There are 10 sweets. How many people can have 2 sweets each?	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects into groups or share equally and see how much is left over.  	Draw dots and group them to divide an amount and clearly show a remainder.  Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 13 ÷ 4 = 3 r1  As knowledge of place value improves, children can begin to jump in multiples of 10: 92 ÷ 3 = 30 r2 	Children use knowledge of times table facts to quickly calculate divisions involving remainders. For example: 27 ÷ 5 = 5 r2 Go on to combining knowledge of times tables with place value to calculate more difficult divisions. For example: 137 ÷ 4 = 34 r1

Short division ('bus stop')	 Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look at how much is in 1 group so the answer is 14.	Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. $\begin{array}{r} 1\ \ 8 \\ 4\overline{)\ 7\ {}^{3}2} \end{array}$ Move onto divisions with a remainder. $\begin{array}{r} 1\ 9\ r3 \\ 4\overline{)\ 7\ {}^{3}9} \end{array}$ This can also be recorded as a fraction: 19 ¾ Finally move into decimal places to divide the total accurately. $\begin{array}{r} 1\ \ 9\ .\ 7\ \ 5 \\ 4\overline{)\ 7\ \ {}^{3}9.{}^{3}0\ {}^{2}0} \end{array}$ $\begin{array}{r} 1\ 4\ .\ 6 \\ 35\overline{)\ \underline{5\ \ 1}\ {}^{16}1.{}^{21}0} \end{array}$

<table>
<tr>
<td>Long division

NB: some children may prefer to use a ‘short division’ method for ‘long division’ problems; they should have both modelled and choose the method that is most efficient for them.</td>
<td>


$$\begin{array}{r} 23\text{ R.}2 \\ 3\overline{)71} \\ -60 \\ \hline 11 \\ -9 \\ \hline 2 \end{array}$$

Using dienes or place value counters, we start with 7 tens and 1 one, to be divided into 3 groups. We can put 2 tens in each group, so we write a 2 in the tens column. In all, we've put 6 tens into the groups (3 x 2 tens), so we write 6 tens (60) below. We are left with 11 (1 ten and 1 one). We will need to exchange the ten for 10 ones so we can put 3 ones in each group (using 9 ones in all), and we will have a remainder of 2.
</td>
<td></td>
<td>
432 ÷ 15 becomes

$$\begin{array}{r} 28\text{ r }12 \\ 15\overline{)432} \\ 300 \\ \hline 132 \\ 120 \\ \hline 12 \end{array}$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$\begin{array}{rl} 28 & \\ 15\overline{)432} & \\ 300 & 15\times20 \\ \hline 132 & \\ 120 & 15\times8 \\ \hline 12 & \end{array}$$

Answer: $28\frac{4}{5}$

$\frac{12}{15} = \frac{4}{5}$

432 ÷ 15 becomes

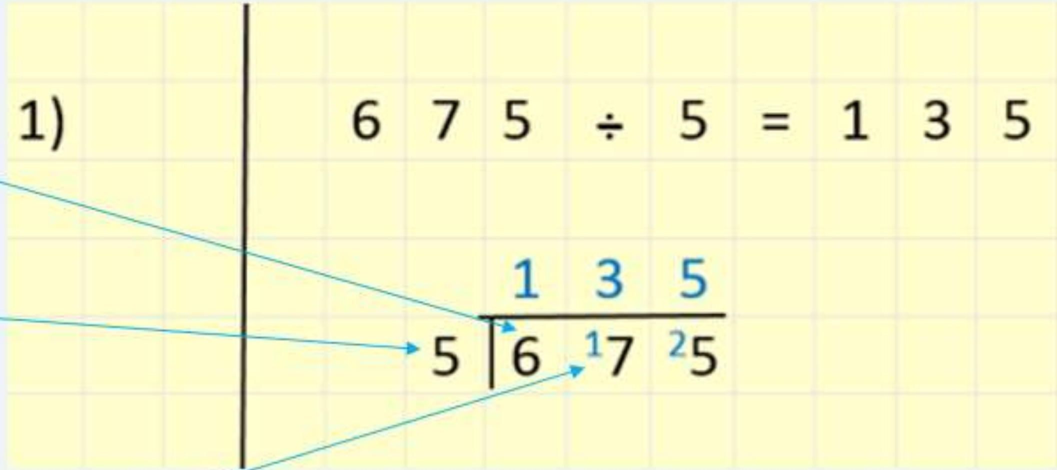
$$\begin{array}{r} 28{\cdot}8 \\ 15\overline{)432{\cdot}0} \\ 30\phantom{0{\cdot}0} \\ \hline 132\phantom{{\cdot}0} \\ 120\phantom{{\cdot}0} \\ \hline 120 \\ 120 \\ \hline 0 \end{array}$$

Answer: 28·8
</td>
</tr>
</table>

Short division ('bus stop')

Expected layout

- ✓ One digit in each box
- ✓ Work from the **left**
- ✓ Dividend (number being divided) inside 'bus stop'
- ✓ Divisor (dividing number) outside 'bus stop'
- ✓ Quotient (answer) on top of 'bus stop'
- ✓ Remainders are 'carried' to next column



Long division

Expected layout

This method works well for solving large division problems, e.g. HTU ÷ TU and ThHTU ÷ TU.

a) Draw the 'Bus Shelter.' Put the number being divided (the dividend) inside the 'Bus Shelter', with the dividing number (the divisor) outside to the left, in this case 432÷15.

b) Now I work from left to right to find the 'goes intos', e.g. "How many 15s go into 4?" Put the answer above the 'Bus Shelter' (in this case it's 0, so we continue and ask, "How many 15s go into 43?" because any remainders are carried onto the next number to make it into a two-digit number). It may be useful to make some jottings at the side; in this case, the 15 times table. This shows that there are 2 15s in 43, which make 30, so I write 2 above the 'Bus Shelter' as the next part of the answer.

c) Now subtract the 30 from 43, using the Column Method of subtraction - this gives me 13 which I write underneath (see example).

d) Now I bring down the remaining 2 from inside the 'Bus Shelter' to make the number 132 (see example).

e) "How many 15s go into 132?" I continue making jottings at the side to help me work this out, writing down the 15 times table. 8 x 15 = 120, so I write 8 above the 'Bus Shelter' as the next part of the answer.

f) Now I subtract the 120 from 132, again using the Column Method of subtraction - this gives me 12 which I write underneath (see example).

g) "How many 15s go into 12?" I can't solve this, and I have no more digits left to bring down, so 12 becomes the remainder. The answer is 28r12.